(7 pages) **Reg. No. :** ....

Code No. : 6843 **Sub. Code : PMAE 23**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Second Semester

Mathematics

ELECTIVE - PARTIAL DIFFERENTIAL EQUATIONS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The primitive of the equation $(x^2z - y^3)dx + 3xy^2dy + x^3dz = 0$ is

   (a) $xz + y^3 = cx$ (b) $x^2z + y^3 = c$

   (c) $x^2z + y^3 = cx$ (d) $x^2z + y^2 = cx$

2. If X is a vector such that $X \cdot Curl X = 0$ and μ is an arbitrary function of x, y, z, then $(\mu X) \cdot Curl(\mu X) =$ ___________.

   (a) μ (b) 0

   (c) 1 (d) X

3. By eliminating the arbitrary constants from $2z = (ax + y)^2 + b$ we get the partial differential equation ___________.

(a) $px + qy = q$ (b) $px + qy = 0$

(c) $py + qx = q^2$ (d) $px + qy = q^2$

4. The equation $z = f(x - y)$ gives the partial differential equation ___________.

(a) $p + q = 0$ (b) $p - q = 0$

(c) $px + qy = 0$ (d) $py - qx = 0$

5. Along every characteristic strip of the equation $F(x, y, z, p, q) = 0$, the function $F(x, y, z, p, q)$ is ___________.

(a) zero (b) a constant

(c) independent (d) equal

6. If every solution of $f(x, y, z, p, q) = 0$ is also a solution of $g(x, y, z, p, q) = 0$, then they are said to be ___________.

(a) equal (b) equivalent

(c) compatible (d) solvable

7. $F(D, D')e^{ax+by}$ = ___________ e^{ax+by} .

(a) $\frac{1}{F(a, b)}$ (b) $F(a, b)$

(c) $F(x+a, y+b)$ (d) $F(b, a)$

8. $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$ is called the one-dimensional ___________ equation.

(a) diffusion (b) heat

(c) wave (d) harmonic

9. The characteristic cone at a point touches ___________ characteristic surface(s) at the point.

(a) only one (b) at least one

(c) many (d) all the above

10. The surface generated by the bicharacteristics of the linear equation $L(u) = 0$ is called a ___________.

(a) Monoid (b) Characteristic cone

(c) Conoid (d) Conic

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Verify that the equation $x(y^2 - a^2)dx + y(x^2 - z^2)dy - z(y^2 - a^2)dz = 0$ is integrable and solve it.

Or

(b) Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.

12. (a) Find the general integral of $z(xp - yq) = y^2 - x^2$.

Or

(b) Prove that the general solution of the linear partial differential equation $Pp + Qq = R$ is $F(u, v) = 0$ where F is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ form a solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

13. (a) Show that the equations $xp - yq = x$, $x^2 q + q = xz$ are compatible and find their solution.

Or

(b) Find a complete integral of the equation $p^2 x + q^2 y = z$.

14. (a) Explain the role of second order equations in Physics.

Or

(b) Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}.$$

15. (a) Classify the equation $u_{xx} + u_{yy} = u_z$.

Or

(b) Classify the equation $u_{xx} + u_{yy} = u_{zz}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that a necessary and sufficient condition that the Pfaffian Differential equation $X \cdot dr = 0$ should be integrable is that $X \cdot Curl X = 0$.

Or

(b) Verify that the equation $z(z + y^2)dx + z(z + x^2)dy - xy(x + y)dz = 0$ Is integrable and find its primitive.

17. (a) If u is a function of x, y and z which satisfies the equation

$$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$$ show that u contains x, y and z only in combinations $x+y+z$ and $x^2+y^2+z^2$.

Or

(b) Find the equation of the integral surface of the differential equation $2y(z-3)p+(2x-z)q=y(2x-3)$ which passes through the circle $z=0$, $x^2+y^2=2x$.

18. (a) Find the solution of the equation $z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ which passes through the x-axis.

Or

(b) Explain Charpit's method.

19. (a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.

Or

(b) Explain the method of solving reducible equations.

20. (a) Discuss the characteristics of equations in three variables.

Or

(b) Explain the method of separation of variables and illustrate it by an example.